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Stable risk-sharing

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Abstract

We analyze the evolution of contract participation and evaluate the selection of risk-sharing contracts in the presence of moral hazard. Organizations specify rules for sharing output among producers, and so affect the extent of private investment in production. Organizations are rigid, as some details of the contract are fixed, but people are free to move around. In the presence of rigidity, equilibrium displays coordination failure because potentially efficient contracts can fail to attract participants. Methods of evolutionary stability are used to select equilibria when organizations compete for members. We identify stable contracts which survive competition against any other. Stable contracts need not be efficient, but for large groups the loss becomes small.

JEL classification: D2; D8; C7

Keywords: Organizations; Risk-sharing; Incentives; Stability; Evolution

1. Introduction

Organizations are sometimes resistant to change but people are free to move around. This paper explores the extent to which the mobility of individuals mitigates the effects of organizational rigidity. Our specific context is that of risk-sharing arrangements. We begin with a fairly standard moral hazard model. Production is risky and depends on privately known investment levels as well as exogenous uncertainty. Individual producers are risk-averse and can agree to share risks with others. Sharing arrangements, or organizations, operate by different rules. They offer different contracts and may have different membership sizes. Individuals choose to participate in one of the competing arrangements and their utility depends on both the contract as well as the membership of the groups. The first idea we bring to this model is of organizational rigidity—the terms of the contract cannot be

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adjusted to attract new members. Given a fixed set of organizations, we ask if people will end up choosing to participate in efficient arrangements. It turns out that equilibrium displays coordination failure as potentially efficient contracts fail to attract participants; competition does not eliminate inefficient contracts. A difficulty with this approach is that every contract can survive competition. This paper provides a novel approach to the selection of contracts using the notion of stochastic stability developed in Foster and Young (1990), Kandori et al. (1993), Young (1993). We specify the dynamics of the process by which individuals choose their organization and examine its long-run steady state to select stable organizations. Stable risk-sharing corresponds to a contract that will eventually attract members away from any competitor. We show that stable contracts exist, and that they may fail to achieve efficient risk-sharing. As a useful mnemonic, stable risk-sharing is only half efficient—the stable contract for a group is efficient for a group half its size.

With risk-sharing in the presence of moral hazard, full insurance will generally not be optimal. Investments, being private information, are not contractible and if incomes are shared *ex-post*, we get underinvestment *ex-ante*. This is the familiar trade off between risk-sharing and incentives. Virtually everything is known about this model at a theoretical level, following Mirrlees (1974), Hölmstrom (1982), Rasmussen (1987). Specifically, that first-best risk-sharing can be achieved (or approximated) for any group of fixed size by the design of appropriate sharing rules. These contracts can be complex, and may involve arbitrarily large punishments for individuals with low output, or randomizing shares of individual partners, or throwing part of output away in some contingencies. Further, as Hölmstrom and Milgrom (1987) have observed, such contracts “perform ideally if the model’s assumptions are precisely met, but can be made to perform quite poorly if small deviations in the assumptions . . . are introduced.” That the contracts are so different from those actually observed in the real world leads them to suggest that simple linear contracts, of the type widely used, are likely to be robust in complex environments, and to develop a model in which a linear compensation scheme turns out be optimal.

We are concerned with a different type of complexity. Within natural classes of contracts, the efficient contract depends crucially on the number of participants, and needs to be rewritten whenever participants enter or leave the organization. In reality, we suspect that organizations, or institutions, are relatively rigid.[1] For our analysis, we associate rigidity with fixed and unchanging rules. Rules are rigid, but individuals are mobile. They can choose which organization to participate in. Are bad rules eliminated by competition between organizations?

It is necessary to start from simple and realistic contracts for risk-sharing. Our choice is motivated by empirical studies, by Gaynor and Gertler (1995), Lang and Gordon (1995). They examine medical and law partnerships respectively. The typical arrangement among partners allows each one to keep a fraction of their revenues, and contribute the rest to a pool to be shared across the partnership. This corresponds to linear, and budget-balancing sharing rules. An efficient rule, within this family, is one where the fraction is chosen to maximize the utility of group members. Different practices may offer different sharing formulae, and partners can quit to join a practice offering, say, a higher share to keep back. These studies follow up on earlier analysis by Gilson and Mnookin (1985), who observed that lawyers’

[1] Most obviously, it is costly to make frequent changes to contracts. There could be lack of unanimous support in cases where change creates winners and losers.

compensation is more equal than their contribution. A very different group of studies, such as Rosenzweig (1988), Townsend (1992) evaluate risk and insurance in village economies. Their findings, which evaluate the extent of consumption-smoothing, find that risks are indeed shared, but only partially. American lawyers and Indian farmers face very different economic environments. The very disparate pieces of evidence suggest that incomplete risk-sharing is a robust phenomenon. Our analysis examines whether evolutionary stability provides an explanation. Equal sharing and zero-sharing are two of the many candidate rules for these risk-sharing arrangements. As it happens, neither is stable.

We need a further element to motivate this approach. As we mention earlier, risk-sharing is more desirable if risks can be spread among many. Specifically, this translates to the fact that risk-sharing organizations have scale economies built in *irrespective* of efficiency. It is easy to see why it is more valuable to join a larger group if risks are shared efficiently. As it happens, scale effects operate even if contracts are not efficient (we demonstrate this formally). In consequence, there are multiple participation equilibria when organizations compete: efficient contracts can emerge as an equilibrium outcome, but so can inefficient ones. Indeed, a fully autarkic outcome, where there are no takers for an efficient contract, is a possible equilibrium precisely because efficiency depends on the participation decisions of others. This is the content of Proposition 1: it may make sense for an individual to join the larger group even if the competing one offers a potentially better contract. Participation choices can lead to coordination failure, for essentially the same reasons as in Diamond (1982), or David (1985). It is natural to ask if some of these equilibria are fragile because an exogenous shock, which affects the size of one organization, can set off a domino effect. An increase in membership of the organization, however achieved, may make it more attractive to potential participants. As more people join, the increase in size further enhances the benefit of participation. With small stochastic shocks, some equilibria are much more likely to emerge in the long run. We apply the criterion of evolutionary stability to formalize this effect. This analysis draws on evolutionary selection, as set out in Kandori et al. (1993), Foster and Young (1990), Young (1993), Blume (1995) and, for contracting in particular, Young and Burke (2001).

An especially appealing feature of evolutionary models, for our application, is their interpretation as models of learning and bounded rationality. It has long been speculated that an understanding of bounded rationality is important for the study of organizations. Here, taking the distribution of choices in the population as given, agents attempt to optimize. They manage to do so with high probability, and revise their decisions only occasionally. This leads to a dynamic process by which beliefs and behavior evolve and settle down at a Nash equilibrium. We consider the competition between contracts (for group members). A contract is stable if it survives in competition with any other (linear and budget-balancing) contract. In Proposition 2, we characterize the stable contract, and show that it need not achieve efficiency. Specifically, the contract is only half-efficient: it is the efficient contract for an organization of half the size.

2. Risk, contracts, and incentives

There are N individual producers, $i = 1, \ldots, N$, who produce output y_i. Output is uncertain, and depends on private investments. Specifically, let $e_i \in [0, 1]$ be the amount invested by individual i. Her output is

$$y_i = e_i^\alpha + \epsilon_i; \quad 0 < \alpha < 1$$

where ϵ_i is a random shock.[2] We assume that this has expectation μ and variance ν, both finite and positive, and is independent across individuals. Producer i incurs total cost $C_i = ce_i$ where c is assumed greater than α.

Individual producers are risk averse. If x_i is the random income of individual i, her utility is

$$V(x_i, e_i) = Ex_i - ce_i - \frac{\rho}{2}\text{var}(x_i).$$

Expectations are taken with respect to the distribution of x_i conditional on e_i. This is the approximate certainty equivalent income of i; the approximation is exact for exponential utility if x_i has a normal distribution. We find it convenient to work with a direct mean–variance representation for utilities, with ρ the common parameter of absolute risk aversion.

Individuals form groups to share production risks. Any such group offers a contract of the form

$$x_i = \sigma y_i + (1-\sigma)\bar{y}; \quad 0 \le \sigma \le 1$$

where $\bar{y}$ is the average output of the group. Individuals keep a fraction σ of their own output, and contribute the rest to a common pool, which is shared equally. Note that $\sigma = 1$ corresponds to no sharing, or autarkic contracts, and $\sigma = 0$ to equal sharing.

A *risk-sharing contract* is fully specified by the sharing rule and by the number of participants who agree to share according to the rule: this is just (σ, M), with $\sigma \in [0, 1]$ and integer M. As potential participants are identical *ex-ante*, further details of membership are irrelevant. The *first best* contract for risk-sharing is easy to characterize. This corresponds to *equal sharing*, i.e. $\sigma = 0$, and *full participation*, i.e. $M = N$. The efficient level of investment is symmetric, and equal to $e^* = (\alpha/c)^{1/(1-\alpha)}$. Risks are efficiently shared by equal division if investment levels are contractible. If so, investment is invariant across group size, and any increase in size is desirable for insurance reasons. We are obviously concerned with situations where this fails.

For any contract (σ, M), define $\xi_M = (1 + (M-1)\sigma)/M$. Incomes are

$$x_i = \frac{1 + (M-1)\sigma}{M} y_i + \frac{1-\sigma}{M}\sum_{j\neq i} y_j; \tag{1}$$

$$= \xi_M y_i + (1-\xi_M)\bar{y}^i_{M-1} \tag{2}$$

where $\bar{y}^i_{M-1}$ is the average output of the group excluding i. In consequence,

$$\boldsymbol{E}x_i = \bar{y}(e_1, \ldots, e_M) = \mu + \xi_M e_i^\alpha + (1-\xi_M)\frac{\sum_{j\neq i} e_j^\alpha}{M-1}; \tag{3}$$

[2] Our results are valid for quite general parametric specifications of this production function. This model is convenient for exposition and for the computations in Section 5.

and

$$\mathrm{var}(x_i) = \nu\left(\xi_M^2 + \frac{(1-\xi_M)^2}{M-1}\right). \tag{4}$$

Payoffs, or expected utilities are

$$V_i(e_1,\dots,e_M;\sigma,M) = \mu + \xi_M e_i^\alpha - ce_i + (1-\xi_M)\frac{\sum_{j\neq i} e_j^\alpha}{M-1} - \frac{\rho\nu}{2}\left(\xi_M^2 + \frac{(1-\xi_M)^2}{M-1}\right) \tag{5}$$

We can deduce the properties of contracts (σ, M) from this specification. In the following, we write $e_i(\sigma, M)$, $\bar{y}_i(\sigma, M)$, and $V_i(\sigma, M)$ for investment, expected output, and expected utilities in the contract (σ, M). All proofs are reported in Appendix A.

Lemma 1 (Underinvestment). *For each* $\sigma \in [0, 1)$, *investment and output levels decrease with membership in the (symmetric) equilibrium*:

$$e_i(\sigma, M+1) = e(\sigma, M+1) < e(\sigma, M); \qquad \bar{y}(\sigma, M+1) < \bar{y}(\sigma, M).$$

Further, $e(\sigma, M) < e^*$ *whenever* $M > 1$.

Investment, as well as expected output and income, decline with membership whenever $\sigma < 1$. Output sharing reduces the incentive to invest, and individuals free-ride. Larger

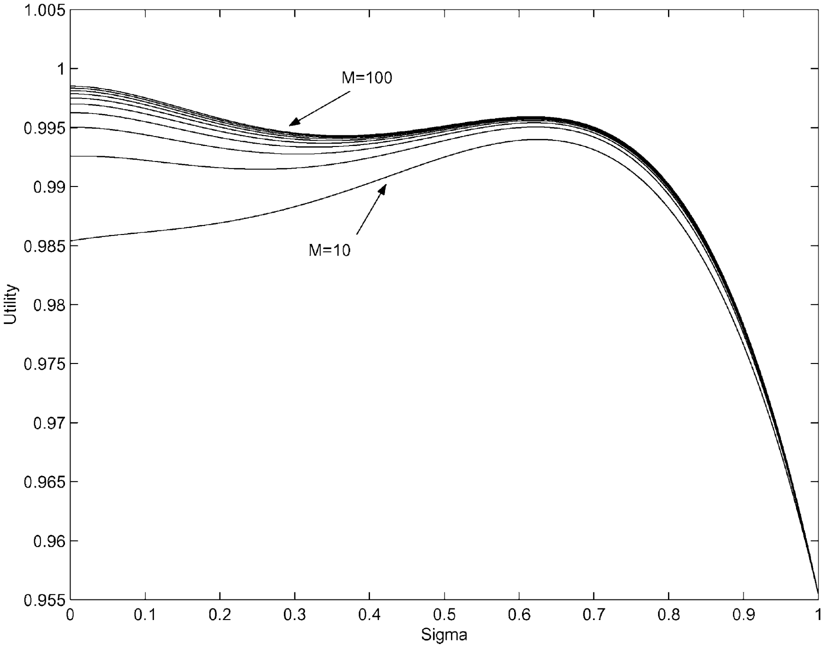

Fig. 1. Utility for different values of M ($\alpha = 0.75$).

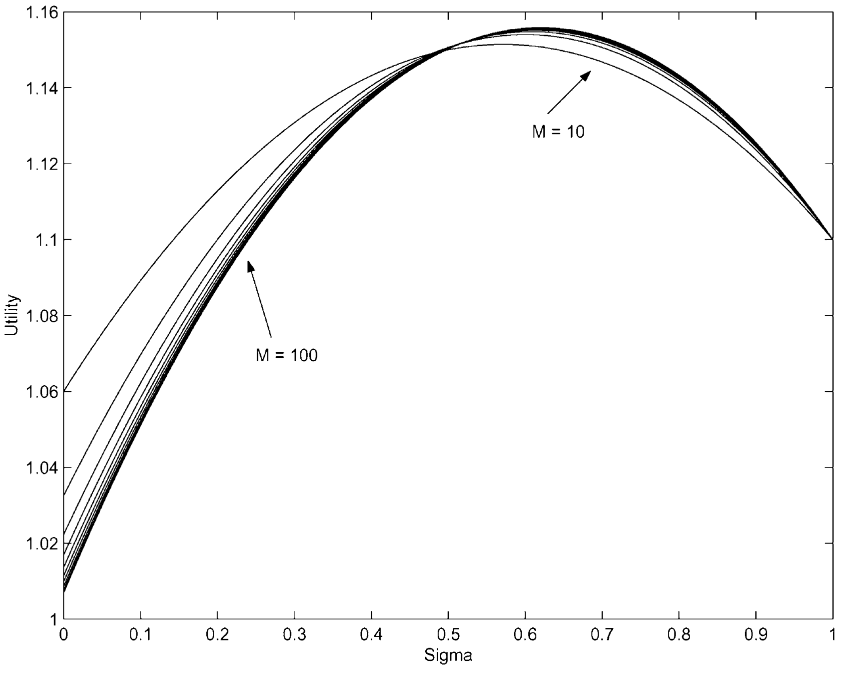

Fig. 2. Utility for different values of M ($\alpha = 0.5$).

groups have more free-riding at equilibrium. This may be more than compensated by the benefit of increased risk-sharing. If the underlying risks are large enough, utility in equilibrium increases with group size.

Lemma 2 (Monotonicity). *For each N, there exists γ_N such that, in equilibrium, for all $\sigma < 1$, $V(\sigma, M)$ increases with M for $M \leq N$ whenever $\rho\nu \geq \gamma_N$.*

Lemma 2 establishes a "sufficient risk" condition. If production is sufficiently risky, all sharing contracts have increasing returns to scale. Some of these may be far from efficient, even within the restricted class of sharing rules. We can alternately think of this as a small N condition.[3] Next we study the properties of different (σ, M). These are illustrated in Figs. 1–6 and, more formally, in Lemmas 3 and 4.

Sharing rules are restricted to be linear, symmetric, and budget-balancing. Our next result shows that groups can tailor their choice of rule, and the efficient sharing rules vary with group size. For each group of size M, the *efficient sharing rule* σ^*_M maximizes payoff $V(\sigma, M)$ with respect to σ. The choice trades off the incentive effect (Lemma 1) with the benefit of risk-sharing.

[3] We note that the monotonicity condition holds globally if α is sufficiently large: γ is invariant to N whenever $\alpha > 1/2$, as shown in the proof.

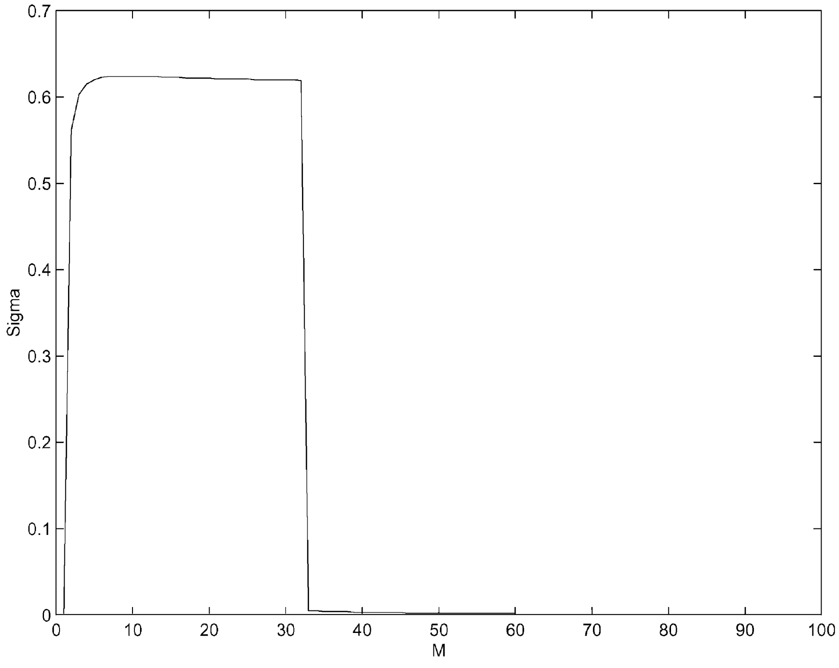

Fig. 3. Optimal sigma of M ($\alpha = 0.75$).

Lemma 3 (Efficient sharing rule). *For each M, an efficient sharing rule σ^*_M exists. Further, $0 < \sigma^*_M < 1$ whenever $M > 1$ and $\rho\nu > 0$. σ^*_M increases with M whenever $\alpha \leq (1/2)$.*

Lemma 3 evaluates the effect of group size on contracts. Any expansion or contraction in the size of a group *should be* matched by a change of contract, to the benefit of participants. There are two cases to consider. If α is small (Figs. 2 and 4), an increase in group size lowers output and the best contract must always raise shares to counter this effect. For larger α (Figs. 1 and 3), the effect can be perverse: larger groups may choose to take advantage of greater insurance at the cost of lower incentives, by decreasing σ^*.

Efficient contracts achieve the best trade off between incentives and risk-sharing. Lemma 4 shows that, when contracts are updated to reflect membership changes, larger groups are always better. In consequence, the best sharing arrangement attainable by a society with N potential participants is (σ^*_N, N).

Lemma 4 (Size and efficiency). *Payoffs increase with size if sharing rules are efficient*:

$$V(\sigma^*_{M+1}, M + 1) > V(\sigma^*_M, M)$$

whenever $\rho\nu > 0$. The contract (σ^*_N, N) dominates all others:

$$V(\sigma^*_N, N) \geq V(\sigma^*_M, M) \geq V(\sigma, M)$$

for any $M < N$ and all $\sigma \in [0, 1]$.

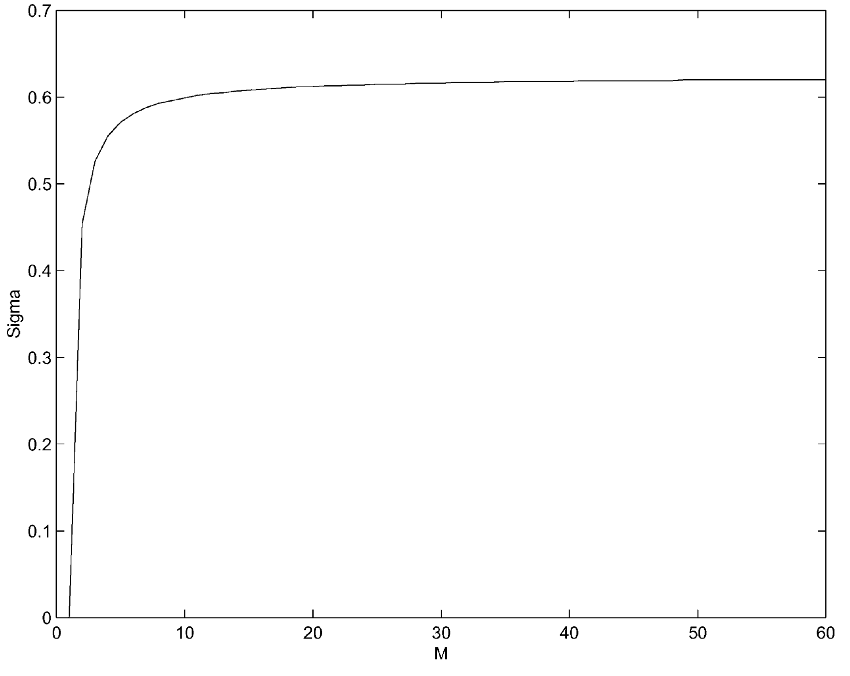

Fig. 4. Optimal sigma of M ($\alpha = 0.5$).

3. Competition and membership

Efficient risk-sharing involves choosing the correct sharing rule for the group. The optimal value of the share parameter, σ, depends upon the size of the group and would need to be changed whenever the membership changed. Here, we take the view that institutions are typically more rigid. Specifically, a group offers a fixed sharing rule, σ, and competes against other groups for membership. Individuals choose whether to join one of the available groups. They may decide not to join any group, in which case they obtain the autarky payoff V_a.[4] Participation decisions determine group membership levels which, in turn, affect the relative attractiveness of the group for others. In this section, we determine the nature of contracts and membership in equilibrium.

Suppose there are $K \geq 1$ groups competing, with sharing rules $\sigma_k \in [0, 1)$ for $k = 1, \ldots, K$. A *participation profile* is an assignment of people to groups and is denoted by a list of non-negative integers $\{M_0, M_1, \ldots, M_K\}$ with $\sum_{k=0}^{K} M_k = N$. M_0 is the number of individuals who choose not to join any of the groups and M_k is the number in group k. We will let $\sigma_0 = 1$ denote the sharing rule of "group" M_0 (everyone keeps all of their own output). Payoffs within a group depend only on its size and not, for instance, on the composition of other groups. When the sufficient risk condition of Lemma 2 is satisfied (i.e. if $\rho\nu \geq \gamma_N$) payoffs will increase with membership. This implies $V(\sigma_k, M_k) \leq V(\sigma_k, N)$

[4] The autarky payoff is the utility $V(\sigma, \cdot)$ from any contract with $\sigma = 1$.

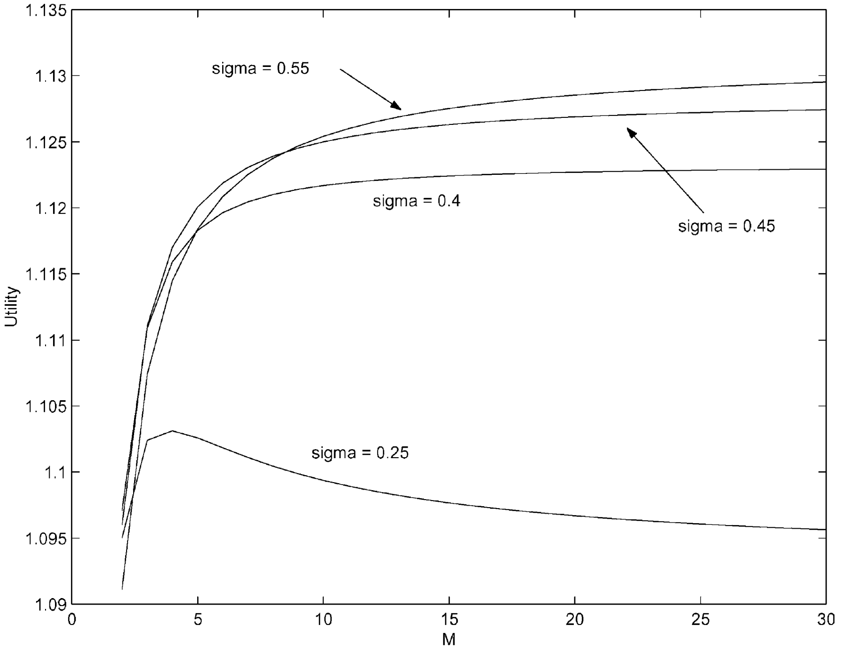

Fig. 5. Utility against M ($\alpha = 0.5$).

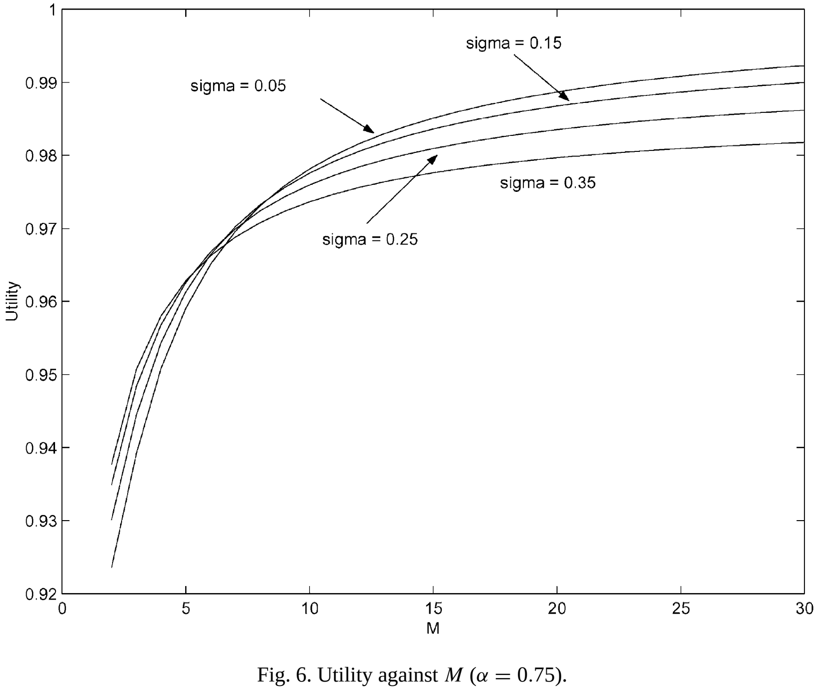

Fig. 6. Utility against M ($\alpha = 0.75$).

for each $k = 1, \dots, K$. The maximal payoffs $V(\sigma_k, N)$ will typically be distinct and can be used to rank the contracts. Without loss of generality, we can choose the indices of the groups such that

$$V(\sigma_1, N) \leq V(\sigma_2, N) \leq \cdots \leq V(\sigma_K, N).$$

In addition, from Lemma 2, $V_a < V(\sigma_k, N)$ for all k. The efficient participation profile is $[0, 0, \dots, 0, N]$. Everyone ought to be assigned to the group which yields the highest payoff at N. Any rearrangement will reduce all payoffs. Since $V(\sigma_K, N) \leq V(\sigma_N^*, N)$ (from Lemma 4) the best possible outcome arises if σ_N^* is among the available contracts. In this case, $\sigma_K = \sigma_N^*$ and everyone should be assigned to the group which offers σ_N^*.

Equilibrium participation levels need not be efficient. We demonstrate this formally by evaluating the equilibrium of a two-stage game where individuals first choose their group and then decide on their investment levels. In the initial stage, there are K groups that offer σ_k ($k \in \{1, \dots K\}$). The participation decision of individual i is indicated by $a_{ik} \in \{0, 1\}$ where $a_{ik} = 1$ means that individual i joins group k, and a_{i0} that she joins none of the groups. Individuals cannot join more than one group, (i.e. $\sum_{k=0}^{K} a_{ik} = 1$). The resulting participation profile is $\{\dots, M_k = \sum_{i=1}^{N} a_{ik}, \dots\}$. In the second stage, individuals choose private investments e_i in response to their contract and the participation profile. Output is produced and shared as specified by the contract (σ_k, M_k). A *participation equilibrium* is a participation profile arising at a subgame perfect Nash equilibrium of this game.

Proposition 1 (Competition and contracts). *Let $\sigma_1, \dots, \sigma_K$ be sharing rules with $0 \leq \sigma_k < 1$, and $V(\sigma_j, N) \leq V(\sigma_{j+1}, N)$ for $j = 1, \dots, K-1$. For each $k = 0, 1, \dots, K$, there exists a participation equilibrium such that $M_k = N$ whenever $V(\sigma_k, N) \geq V_a$. There are no other participation equilibria if $\rho\nu \geq \gamma_N$.*

Proposition 1 displays the nature of co-ordination failure arising from increasing returns to risk-sharing. Every individual joining the same group is a participation equilibrium—irrespective of what the group offers. With competition between K groups there are $K + 1$ equilibria. These are Pareto ranked, as $V_a < V(\sigma_1, N) \leq \cdots V(\sigma_k, N)$. The set of equilibria is insensitive to the efficiency properties of contracts and even the fully efficient contract σ_N^* may fail to attract individuals away from autarky. Increased competition, with a greater set of offerings, merely increases the possible extent of coordination failure.

The failure arises from the fact that a contract must attract many participants in order to succeed, and that relative success feeds back on itself by attracting yet more participants. Similar factors may drive inefficient technology choices (David (1985)), or the volume of trade in markets where individuals search for trading partners (Diamond (1982)). In our framework, equilibria differ in the choice of organizations or contracts. The following quote from Williamson (1995) is pertinent:

> Most of the path dependency literature emphasizes technology (e.g. the QWERTY typewriter keyboard) .... I am not persuaded that technological, as against organizational, path dependency is as important as the literature suggests.

Williamson suggests that inefficient forms of organizations could be established by historical accident, and then may persist. We make the idea operational in the context of a model of risk-sharing, and show how inefficient equilibria could arise. To see how this results in path dependence, we may imagine that σ_N^* is established for a population of size N so that the organization is initially optimal. If the population grows to N', the organization is no longer optimal in the presence of competition from a new organization which offers $\sigma_{N'}^*$. But the organization offering σ_N^* is likely to attract all the new participants.

While historical accidents may sometimes be significant, it is often possible to make more precise predictions about the equilibrium likely to be played. The type of co-ordination failure outlined here can be quite sensitive to perturbations of the kind induced by "mistakes" made by individuals. These mistakes could be genuine errors, or the result of conscious experimentation. A small number of such mistakes can change the relative attractiveness of options to other participants, and set off a domino effect as people find that it is in their interest to move. We turn to this next, and look for stability, or robustness of equilibria to perturbations. Anticipating the results, we find that an autarkic equilibrium is typically not stable. Neither is full insurance ($\sigma = 0$). There always exist stable contracts and we examine their efficiency and other properties.

4. The stability of contracts

Competition does not eliminate inefficient contracts. Proposition 1 shows that full participation in any contract can arise in equilibrium, irrespective of its efficiency. Indeed, autarky—the worst "contract" on offer—can be an equilibrium outcome even though a fully efficient contract is available. Here, we ask if such equilibria are "stable"—in the sense of being robust to perturbations induced by errors in individual choices. In doing this, we begin with situations of pairwise competition, where individuals face a choice between two available contracts at a time—call these σ_k and σ_j.[5] Every now and then, some individual gets an opportunity to revise her participation decision. They evaluate their best decision, given others' choices, but make random errors and choose the wrong decision with a small probability. From this, we deduce the stochastic process guiding participation profiles, and its stationary distribution. The contract (σ_k, N) is *stable* if the stationary distribution puts positive probability on the event $M_k = N$, as the probability of errors in individual decisions vanishes. The stability of a contract is relative—it may be stable in competing against some σ_j, but not others. A contract is *globally stable* if it is stable relative to every $\sigma_j \in [0, 1]$.

We are applying the selection criterion suggested by Kandori et al. (1993), Young (1993). In our case, the transition rule, and stochastic process governing participation follows the birth–death process of Blume (1995).[6] Formally, let $\{\sigma_k, \sigma_j\}$ be the two rules available, and let $\{M_{kt}, M_{jt}\}$ be the participation profile at time t. By construction, $M_{jt} = N - M_{kt}$, so that the state of the system is described entirely by the variable M_{kt}. At any time, some individual gets an opportunity to change her participation decision. They all have the same probability, $1/N$, of a decision revision opportunity. The individual chooses one of the two groups to participate in. She chooses her payoff-maximizing action, or best response, with

[5] One of these could be the contract $\sigma = 1$ equivalent to autarky.

[6] Some of our assumptions differ: the stochastic process on the state variable is within the class studied in Blume (op.cit.).

probability $1 - \epsilon$, and makes the wrong choice with probability ϵ. The state of the system then moves to $M_{k,t+1}$. It is useful to derive the transition probabilities explicitly.

At any time, the two contracts are (σ_k, M_{kt}) and $(\sigma_j, N - M_{kt})$. At time t, a single individual gets the opportunity to revise her decision. This individual is either in the k-contract, with probability M_{kt}/N, or in the j-contract, with probability $(1 - M_{kt}/N)$. A k-contract holder is better off staying in her group if $V(\sigma_k, M_{kt}) \geq V(\sigma_j, N - M_{k,t} + 1)$, and moving otherwise. Similarly, a j-contract holder is better off moving if $V(\sigma_k, M_{kt} + 1) \geq V(\sigma_j, N - M_{kt})$.

We note, first, that

$$M_{k,t+1} \in \{M_{kt} - 1, M_{kt}, M_{kt} + 1\}.$$

The state M_k changes by at most 1 in either direction, because only one individual gets a revision opportunity at any time. We write d_t for the probability of a "death"—the event that $M_{k,t+1} = M_{kt} - 1$. And b_t is the probability of a "birth"—$M_{k,t+1} = M_{k,t} + 1$. These can be deduced as functions of the current state M_{kt}. Let

$$\Delta_{kj}(M) = V(\sigma_k, M) - V(\sigma_j, N - M + 1).$$

A "death" occurs only if an individual in the k-contract gets a revision opportunity. It is their best response whenever $\Delta_{kj}(M_k) < 0$; otherwise, they may move by mistake. It follows that

$$d_t = \begin{cases} \dfrac{M_{kt}}{N}(1-\epsilon) & \text{if } \Delta_{kj}(M_{kt}) < 0 \\ \dfrac{M_{kt}}{N}\epsilon & \text{if } \Delta_{kj}(M_{kt}) \geq 0. \end{cases}$$

Similarly, a "birth" occurs only if an individual in the j-contract gets a revision opportunity; moving is her best response if $\Delta_{kj}(M_k) \geq 0$. As a result,

$$b_t = \begin{cases} \left(1 - \dfrac{M_{kt}}{N}\right)\epsilon & \text{if } \Delta_{kj}(M_{kt}+1) < 0 \\ \left(1 - \dfrac{M_{kt}}{N}\right)(1-\epsilon) & \text{if } \Delta_{kj}(M_{kt}+1) \geq 0. \end{cases}$$

The current state M_{kt} determines $d_t,\ b_t$ completely. We write $d(M_k),\ b(M_k)$ as the functions defined here. The stochastic process defined by these transition probabilities has a unique stationary distribution,

$$\boldsymbol{P}_\epsilon = \left\{ P_\epsilon(m); m = 0, 1, 2, \ldots, N; \sum_{m=0}^{N} P_\epsilon(m) = 1 \right\}$$

The contract (σ_k, N) is stable relative to σ_j if

$$\bar{P}(N) = \lim_{\epsilon \to 0} P_\epsilon(N) > 0.$$

In other words, (σ_k, N) is stable if the state $M_k = N$ is stochastically stable. It is *globally stable* if it is stable relative to each $\sigma_j \in [0, 1]$.

Proposition 2 (Relative stability). *Let $\rho\nu \geq \gamma_N$, and define*

$$\frac{N^+}{2} = \begin{cases} \dfrac{N}{2}+1 & \text{if } N \text{ is even;} \\ \dfrac{N+1}{2} & \text{if } N \text{ is odd.} \end{cases}$$

The contract (σ_k, N) is stable relative to σ_j if, and only if $\Delta_{kj}(N^+/2) \geq 0 \Leftrightarrow$

$$(\boldsymbol{S_k}) \quad V\left(\sigma_k, \frac{N^+}{2}\right) \geq V\left(\sigma_j, N+1-\frac{N^+}{2}\right).$$

We provide a direct proof because the stationary distribution of the birth-death process can be explicitly evaluated and this is desirable. The proposition can also be deduced using Theorem 1 of Ellison (2000). Condition $(\boldsymbol{S_k})$ guarantees, for the sharing rule σ_k, that its radius cannot be smaller than its co-radius. Intuitively, it takes at least as many mistakes to exit the basin of attraction of σ_k as it does to enter it. As a consequence, in the long-run, the organization offering σ_k would be observed for a strictly positive fraction of the time. In our paper, as in Kandori, Mailath & Rob and elsewhere, stable equilibria display the 1/2-dominance property. A symmetric equilibrium is stable if the strategy is an optimal response whenever at least half the population plays it.

The condition $\rho\nu \geq \gamma_N$ is used to guarantee that payoffs are strictly monotone in participation (Lemma 2). This results in a "Total Bandwagon Property" of the kind set out in Kandori and Rob (1998). It is possible to get interesting dynamics even when this assumption is violated. We study an example later in this section.[7]

Finally, condition $(\boldsymbol{S_k})$ is necessary and sufficient for the stability of (σ_k, N). The corresponding condition for the stability of (σ_j, N) can be written as

$$(\boldsymbol{S_j}) \quad V\left(\sigma_j, \frac{N^+}{2}\right) \geq V\left(\sigma_k, N+1-\frac{N^+}{2}\right).$$

It is possible for both conditions to hold. If so, both contracts can prevail, with positive probability in the long-run equilibrium. Stable or long-run equilibria display co-ordination failure of the kind demonstrated in Proposition 1. This failure is smaller, because very inefficient contracts cannot be stable.

Corollary 1. *Let $\rho\nu \geq \gamma n$. The contract (σ_k, N) is stable relative to σ_j whenever $V(\sigma_k, M) \geq V(\sigma_j, M)$ for $M = 1, 2, \ldots, N$.*

We note that $(N^+/2) \geq N+1-(N^+/2)$, and $V(\sigma_k, N^+/2) \geq V(\sigma_j, N^+/2)$ implies $(\boldsymbol{S_k})$. Corollary 1 allows us to examine the case of risk-sharing *without* moral hazard. In this case, the $\sigma = 0$ organization is optimal for all M and will always be stable. The same is true for the perfectly flexible organization which chooses σ^*_M for all M.

[7] This property is useful in making the transition from stable rules ($\bar{P}(0) < 1$) to stable contracts ($\bar{P}(N) > 0$). With the bandwagon property, one implies the other.

Corollary 2. *Let $\rho\nu \geq \gamma n$. Autarky is not stable relative to $\sigma_j < 1$ whenever $N \geq 3$.*

The payoff to autarky (or $\sigma_j = 1$) is V_a irrespective of M_j. For any non-trivial sharing rule, $V(\sigma, M) > V_a$ for $M \geq 2$.

Corollary 3. *Stable contracts need not be efficient.*

Stability requires that σ_k dominate at $N^+/2$. Efficiency requires dominance at N. We can always find a pair of sharing rules satisfying

$$V\left(\sigma_k, \frac{N^+}{2}\right) > V\left(\sigma_j, \frac{N^+}{2}\right); \quad V(\sigma_k, N) < V(\sigma_j, N).$$

It suffices to choose $\sigma_j = \sigma^*_N$ and $\sigma_k = \sigma^*_{N^+/2}$.

It is possible to provide some simple intuition.[8] In pure coordination games, the Pareto dominant equilibrium is stochastically stable (Kandori and Rob, 1998). This makes our finding in Corollary 3 somewhat surprising. The fundamental difference between this framework and previous application to population games with random matching lies in the nonlinearity of payoffs with respect to M_k, the number of players adopting strategy k. Suppose payoffs $V(\sigma_k, M_k)$ were linear and increasing, with $V(\sigma_k, 1) = V(\sigma_j, 1) = V_a$. Then efficiency, i.e. $V(\sigma_k, N) > V(\sigma_j, N)$, would imply stability, $V(\sigma_k, N/2) > V(\sigma_j, N/2)$. Payoffs in our model, derived from economic fundamentals, are monotone but *not* linear. Figs. 5 and 6 describe how payoffs in equilibrium change with M and it is easy to see how one contract may dominate another at some values of M, whereas the opposite is true at other values.

Proposition 3. *Let $\rho\nu \geq \gamma_N$. The contract $(\sigma^*_{N^+/2}, N)$ is globally stable. This is the unique globally stable contract if N is odd.*

Finally, Proposition 3 shows that the globally stable contract is unique whenever N is odd. In this case, the efficient contract (σ^*_N, N) could not possibly be globally stable. With even N, other contracts could be stable: $(\sigma^*_{N/2}, N)$ is always stable. Suppose $N = 2m$. We note that

$$V(\sigma^*_{m+1}, m+1) \geq V(\sigma^*_m, m+1) > V(\sigma^*_m, m) \geq V(\sigma^*_{m+1}, m).$$

The stability conditions write as

$$(\boldsymbol{S_{m+1}})\ V(\sigma^*_{m+1}, m+1) \geq V(\sigma^*_m, m)$$

$$(\boldsymbol{S_m})\ V(\sigma^*_m, m+1) \geq V(\sigma^*_{m+1}, m).$$

Both are satisfied by construction.[9] The losses (from the use of stable rather than efficient contracts) become negligible as N becomes large.

[8] See also Kim (1996) who studies equilibrium selection and efficiency in a number of n-person coordination games.

[9] For even N, the efficient contract, (σ^*_N, N), may also be stable. Say (σ^*_N, N) is stable in addition to $(\sigma^*_{N^+/2}, N)$. In the case where these two contracts compete, both would be seen for a positive fraction of the time in the long-run (the system switching randomly between the two organizations).

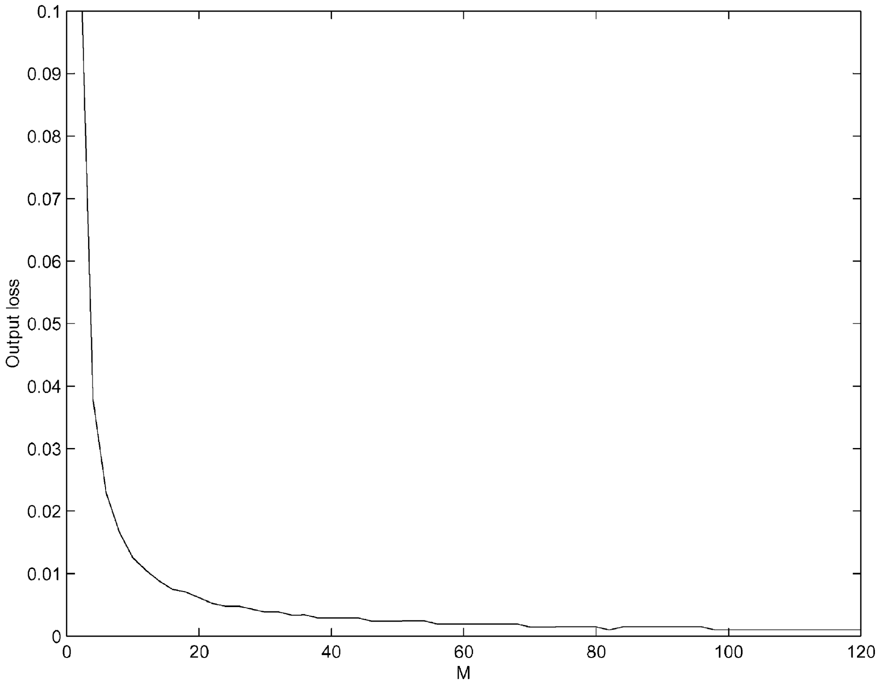

Fig. 7. Output loss (stable vs. efficient contract, $\alpha = 0.5$).

Corollary 4 (Asymptotic efficiency). *The utility loss is small for large groups*:

$$\left| V(\sigma^*_{N+/2}, N) - V(\sigma^*_N, N) \right| \to 0 \; as \; N \to \infty.$$

The proof follows from the fact that the σ^*_N converge: $(\exists\sigma^*)\,[\sigma^*_N \to \sigma^*]$, whereby $\sigma^*_{N+/2} \to \sigma^*$. Hence, $\sigma^*_{N+/2} \to \sigma^*_N$. This result can also be seen in the numerical simulations reported in Figs. 7–10.

The restriction to pairwise contests turns out to be inessential. The final proposition of this section extends our results to the case of competition between K groups. The state of the dynamic system is now a participation profile $(M_1, \dots, M_K)$. The birth and death probabilities are defined in the proof; once again the process has a unique stationary distribution denoted by P_ϵ. Let s_k denote the profile with $M_k = N$. A contract (σ_k, N) is said to be stable (relative to contracts with indices in $K - \{k\}$) if

$$\bar{P}(s_k) = \lim_{\epsilon \to 0} P_\epsilon(s_k) > 0.$$

In other words, (σ_k, N) is stable if the state s_k is stochastically stable. A contract is globally stable if it is stable in every K-contest.

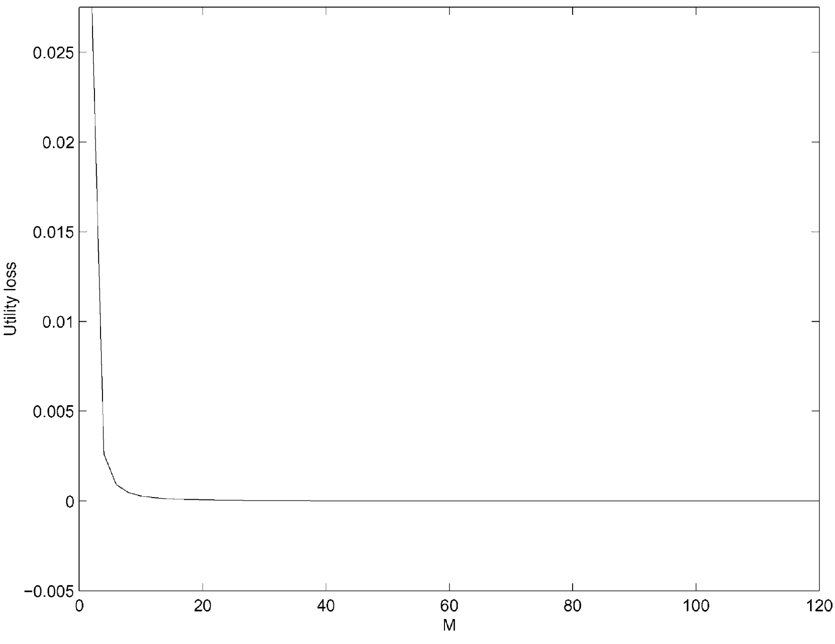

Fig. 8. Utility loss (stable vs. efficient contract, $\alpha = 0.5$).

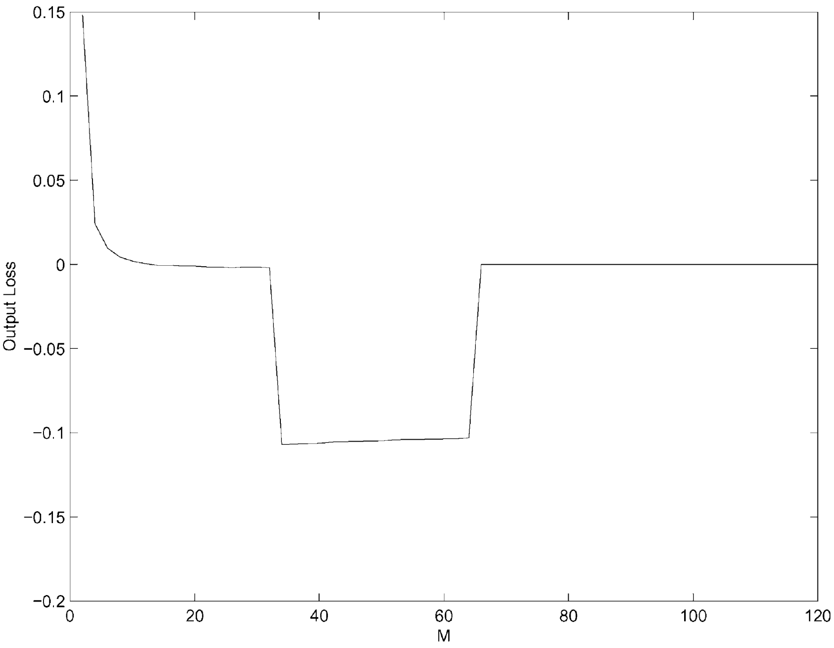

Fig. 9. Output loss (stable vs. efficient contract, $\alpha = 0.75$).

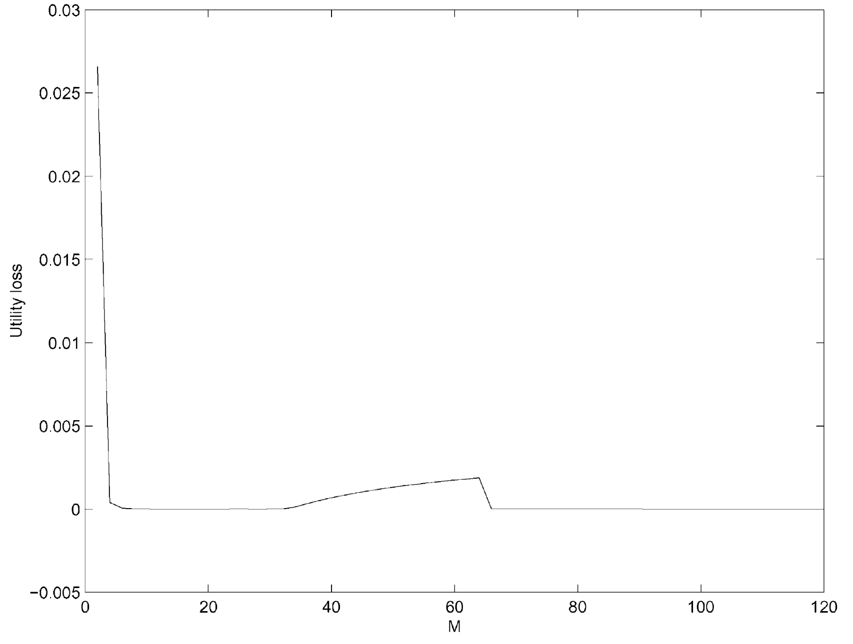

Fig. 10. Utility loss (stable vs. efficient contract, $\alpha = 0.75$).

Proposition 4. *Let $\rho\nu \geq \gamma_N$ and suppose there are $K \geq 1$ groups competing, with sharing rules $\sigma_i \in [0, 1)$ for $i = 1, \ldots, K$. Then,*

$$(\forall i)\left[V\left(\sigma_k, \frac{N^+}{2}\right) \geq V\left(\sigma_i, N+1-\frac{N^+}{2}\right)\right] \Rightarrow (\sigma_k, N) \text{ is stable.}$$

$\sigma^*_{N^+/2}$ *is globally stable.*

Harrington (1999) suggests that rigidity may itself confer an advantage to people, and then spread by imitation. Our last corollary explores this idea for organizations. What happens if at a very small cost a firm can maintain flexibility? Specifically, a flexible group incurs a per period cost of δ (shared by all members equally) for the ability to rewrite contracts. For simplicity, let N be odd. The payoff from belonging to the flexible group is then

$$V_F(M) = V(\sigma^*_M, M) - \frac{\delta}{M}.$$

Note that $V_F(N^+/2) < V(\sigma^*_{N^+/2}, N^+/2)$. As a result we get Corollary 5.

Corollary 5 (Costly Contracting). *For every $\delta > 0$ the flexible organization, with payoffs given by V_F, is not globally stable if N is odd.*

In Propositions 1–4, we use the monotonicity condition $\rho\nu \geq \gamma_N$. When this condition fails, an arrangement may have an efficient size $M^* < N$. The next example explores a situation of this type. We note here that the stable outcome assigns positive probability to $M_k < N$: different individuals may belong to different groups.

4.1. Failure of monotonicity

In the previous analysis, we assume a sufficient risk condition that ensures that payoffs are monotone in M. Absent this condition efficient and stable distributions are likely to be non-degenerate and distinct. The next example demonstrates this.

Suppose that $N = 4$ and that $M^*(\sigma_k) = 2$. We want to evaluate stable outcomes when two groups compete, and both offer σ_k. The efficient outcome is $M_k = 2$ where each group has half the population. It is easy to construct examples with

$$V_k(2) > V_k(3) > V_k(4) > V_k(1) \equiv V_a.$$

We can calculate birth and death rates explicitly: $b_0 = \epsilon$; $d_1 = (1-\epsilon)/4$; $b_1 = 3(1-\epsilon)/4$; $d_2 = \epsilon/2$; $b_2 = \epsilon/2$; $d_3 = 3(1-\epsilon)/4$; $b_3 = (1-\epsilon)/4$; $d_4 = \epsilon$. From the balance conditions $P(n) = P(0)\prod_{j=1}^{n} b_{n-1}/d_n$,

$$P_\epsilon(1) = P_\epsilon(0)4\frac{\epsilon}{1-\epsilon}, \quad P_\epsilon(2) = P_\epsilon(0)6, \quad P_\epsilon(3) = P_\epsilon(0)4\frac{\epsilon}{1-\epsilon}, \quad P_\epsilon(4)=P_\epsilon(0)$$

We obtain $\bar{P} = \lim_{\epsilon\to 0} P_\epsilon$ as

$$\bar{P} = \left[\frac{1}{8}, 0, \frac{3}{4}, 0, \frac{1}{8}\right].$$

Note that the efficient participation profile has weight 3/4 but the stable equilibrium may have overcrowding as all people join one or the other group.

5. Simulations

In this section, we report some numerical simulations on the nature of risk-sharing contracts, including efficiency and stability. We begin by showing how changes in the parameters of the contracting problem affect efficient contracts and report on the loss in output and utility from the of stable rather than efficient contracts. Finally, we examine the dynamics of organizational competition in simulations.

5.1. Evaluating the inefficiency of contracts

In Table 1, we report results for simulations with $\mu = 1$, $\nu = 0.1$, $\rho = 5$ and $c = 1$. The differences between stable and efficient contracts are reported in columns 1 and 2 for $\alpha = 0.25$, 0.5 and 0.75. For small N (less than 10), the difference between stable and efficient sharing rules can be quite large. For example, for $\alpha = 0.75$ and $N = 7$ the optimal contract dictates that participants should keep less than 6% of their own output and share the remainder with others. At the stable equilibrium sharing rule, they keep a much higher proportion, over 23%, of their output. Interestingly, the direction of inefficiency switches

Table 1
Stable vs. efficient contracts

Size (N)	Stable rule ($\sigma^*_{N+/2}$)	Optimal rule (σ^*_N)	Utility loss ($V(\sigma^*_N) - V(\sigma^*_{N+/2})$)	Output loss ($\bar{y}(\sigma^*_N) - \bar{y}(\sigma^*_{N+/2})$)
$\alpha = 0.25$				
3	0.226	0.300	0.002	0.016
5	0.300	0.354	0.001	0.015
7	0.334	0.376	0.0008	0.013
9	0.354	0.388	0.0005	0.011
$\alpha = 0.50$				
3	0.3346	0.401	0.001	0.022
5	0.401	0.445	0.0007	0.017
7	0.430	0.463	0.0003	0.014
9	0.445	0.472	0.0003	0.012
$\alpha = 0.75$				
3	0.375	0.354	0.00006	−0.006
5	0.354	0.231	0.0011	−0.024
7	0.231	0.055	0.0024	−0.022
9	0.159	0.033	0.00164	−0.005

$\rho = 5$, $\nu = 0.10$ for all simulations.

for $\alpha < 0.5$. For example, at $\alpha = 0.25$, the stable contract achieves excessive risk-sharing relative to the efficient one. And so, output may be "too high" at equilibrium contracts if $\alpha > 0.5$, as reported in the last column. We also notice that utility losses, measured in output equivalent terms, are very small and also that they are non-monotone with respect to N. Our further simulations explore these findings.

5.1.1. *Sharing rules and size effects*

The base case simulations assume $\mu = 1$, $\nu = 0.1$, $\rho = 3$ and $c = 1$. We report simulations for $\alpha = 0.75$ and 0.5, respectively. Simulation results are reported in Figs. 1–12. Fig. 1 contains plots of equilibrium payoffs for different contracts (σ, M). Specifically, we plot $V(\cdot, M)$ for various values of M. For low values of M (e.g. $M = 10$), there is a unique maximum point ($\sigma^*_M \approx 0.62$); as M increases, a lower mode emerges, with values of σ^*_M near zero—clearly perceptible at $M = 100$. The payoff function $V(., M)$ is non-convex when $\alpha > 1/2$, and the interior optimum jumps down.[10] As Fig. 3 illustrates, this change occurs around $M = 30$.

In Fig. 2 we set $\alpha = 0.5$, which ensures that $V(\cdot, M)$ is strictly concave. For low values of σ, the graph for $M = 10$ starts off above the other graphs, and then falls below. This shows, for instance, that with equal sharing ($\sigma = 0$) utility actually decreases when M is increased. In this case, we also see (in Fig. 4) that the maximum point increases with M.

In Figs. 5 and 6, we see how utility changes with M when σ is held constant. In the case of $\alpha = 0.75$, we find that $V(\sigma, \cdot)$ increases with M. The interesting property is that for low values of M, an inefficient contract (e.g. $\sigma = 0.25$) actually does better than a contract (such

[10] We note here that randomization in contracts may be desirable: the restriction to deterministic sharing rules binds.

as $\sigma = 0.05$) which is more efficient for the population size of $M = 30$. This property is preserved in the case of $\alpha = 0.5$, only now utility is higher at $M = N$ for higher values of σ. We also see here how, for low values of σ, the optimal size could be smaller than N.

5.2. *Stable versus efficient contracts*

In Figs. 7–10, we plot the output and utility loss for the stable contract compared to the efficient contract. For each M, we compute the investment when the contract $(\sigma^*_{M^+/2}, M)$ is offered. This allows us to compute the output which can be compared with the output from (σ^*_M, M). Utility loss is found by comparing $V(\sigma^*_{M^+/2}, M)$ with $V(\sigma^*_M, M)$. For $\alpha = 0.5$, these losses become smaller with M. This is also the case when $\alpha = 0.75$, except near the singularity. Table 1 also gives a detailed comparison of the stable and efficient contracts.

We notice that $V(\sigma, M)$ is not a concave function of σ when $\alpha = 0.75$ (Fig. 1). This leads to a discontinuity in σ^*_M (Fig. 3) as well as in the output and utility losses relative to the stable contract (Figs. 9 and 10).

5.3. *Competition and evolution*

Finally, Figs. 11 and 12 depict the dynamics of organizational competition. We begin with a random assignment of individuals to the two groups (one offers the efficient contract, the

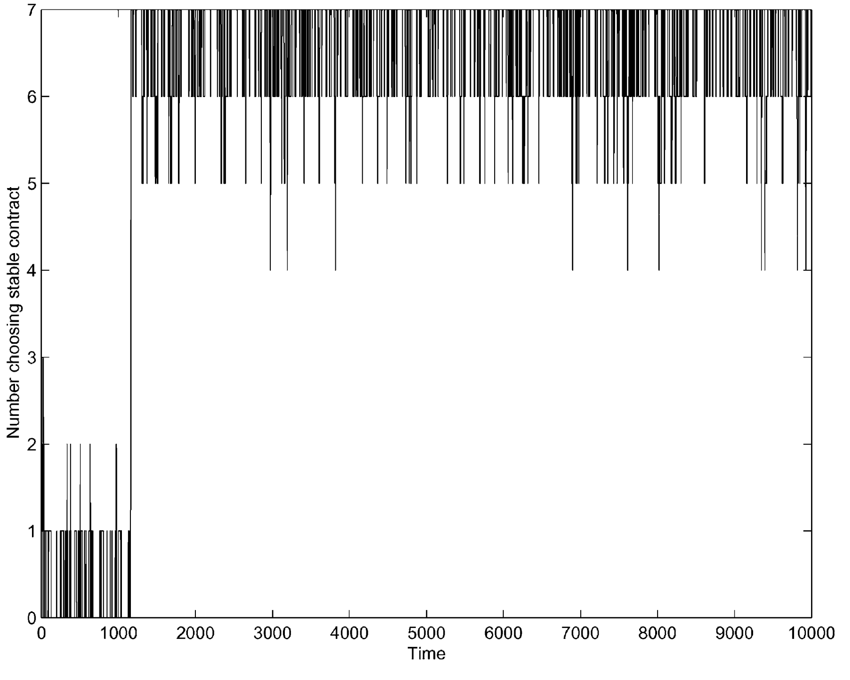

Fig. 11. Competition between stable and optimal contract.

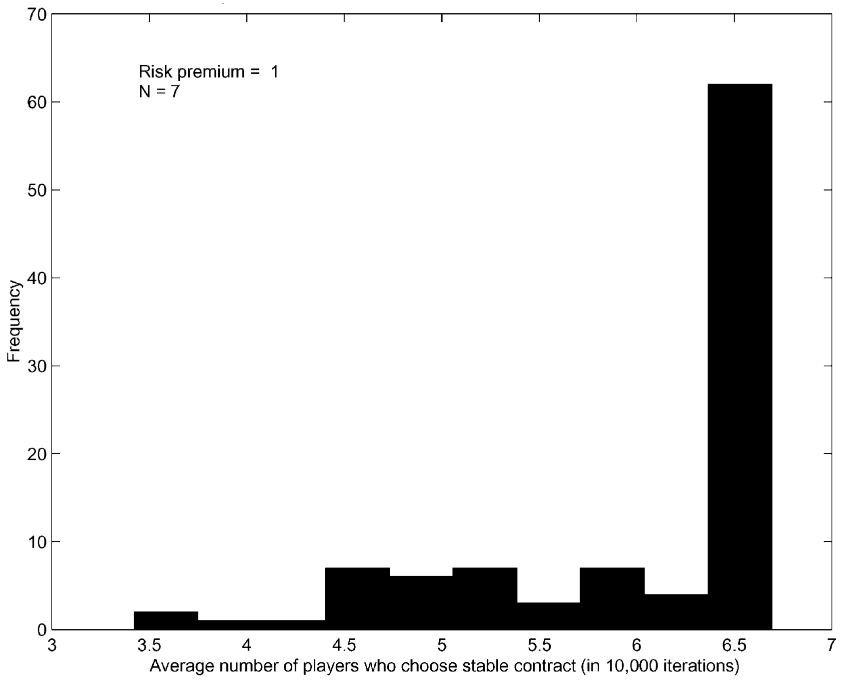

Fig. 12. Competition between the stable and optimal contract.

other offers the stable contract). Each individual gets to revise his participation decision at randomly generated decision opportunities and is then locked into the decision until the next opportunity arises (the intervals are exponentially distributed). When making a decision, players assume that the profile of choices in the population will remain fixed. We show in Fig. 11 how the participation in the stable group evolves with time. The system is in the efficient steady state at the beginning, but soon makes a transition to the stable steady state and stays there for the duration of the simulation. This is typical. We repeat this kind of simulation several times (100 repetitions) and compute the average size of the stable group in a simulation. The frequency of this average is presented in Fig. 12, and this points towards the selection of the stable organization.

6. Conclusions

The possibilities for sharing risk may be limited by moral hazard. Efficiency in such circumstances requires that individuals enter into complex contracts, which depend on subtle informational properties of the distribution of output. In this paper, we consider a situation where the potential set of contracts is simple, and evaluate whether efficient contracts coincide with stable ones. Our main result is that they need not. Contractual efficiency depends on the number of participants. As we show, the nature of inefficiency arises from "overcrowding"—a contract appropriate for (say) 10 players tends to attract twice as many. Put another way, the contract which is right for 20 cannot eliminate it in competition.

We started from the viewpoint that organizations are "rigid" in the precise sense that they do not change their rules of operation. Individuals can move between organizations. Unfortunately, this is not enough to deduce the survival of the efficient organization: indeed as our Proposition 1 shows *any* contract can arise, survive, and attract 100% participation at a subgame perfect Nash equilibrium. The difficulty arises because of scale economies in the nature of institutions. It is individually rational to participate in large organizations irrespective of their efficiency. It is of course possible to stop there, and take the view that equilibrium does not pin down institutional structure, or to assume there are decreasing returns due to further costs of operation not in the model. We take a third point of view, that the structure of institutions or contracts or markets predicted by economic theory should be robust to small mistakes made by participants.

Applying a fairly natural notion of mistakes borrowed from the evolutionary game theory literature, we find that a stable contract is essentially unique. It is, however, inefficient. The inefficiency stems from the nonlinearity of payoffs with size. Interestingly, a lot of inefficient outcomes are ruled out: in particular, the autarkic outcome never survives; full insurance rarely does. When the size of the population becomes large, the loss from the use of stable contracts becomes negligible. We draw attention to this because the potential losses from coordination failure could explode as N increases—this is not true at stable outcomes.

There remains the question of making the menu of available contracts endogeneous. This would require a substantial extension of this work. At the same time, our findings do make some clear predictions. Suppose that we start from a situation where all particants are in some organization, i.e. the state is (σ, N). A competing contract will only survive if it is stable relative to σ. If introducing new contracts is costly we would expect the relative stability condition, $\boldsymbol{S_k}$, to be satisfied successively with strict inequality. The globally stable contract would be immune to such competition (but if the introduction of contracts is costly several other contracts may be immune as well).

We restrict attention to linear, budget balancing contracts. These can be thought of as insurance contracts with deductibles indexed by σ, traded at actuarially fair prices. This interpretation is valid for large N, which suggests that the approach may be useful in understanding the stability of asset structures, especially if profit maximizing firms design the insurance contracts.

There are other approaches to studying incomplete risk-sharing. For example, Kimball (1988), Coate and Ravaillon (1993) evaluate the extent to which repeated interaction may sustain some trade, and explain partial risk-sharing. It is an open question whether that approach is likely to provide a better explanation of observed outcomes.

We begin by imagining that groups operate by different rules, and compete for participants. The question is whether competition eliminates inefficient contracts. Hart (1983) asks the same question about competition between inefficient firms, and finds that competition improves efficiency. Our "firms," or risk-sharing groups, have the property that the relative value of participation increases with its size, and this induces coordination failures. Such a phenomenon arises for many economic institutions, including markets. This is true for markets with costly search (Diamond, 1982), of markets competing against informal arrangements of reciprocity (Kranton, 1996), and the use of money as a medium of exchange (Ritter, 1995).

We assume that institutions are rigid, but do not attempt to explain why. It is possible that rigid institutions are themselves a product of evolution. Harrington (1999) suggests that rigid behavior by individuals may do well in competitive environments, and may spread by imitation. For our framework, this is a further illustration that stable rules are not necessarily efficient.

Acknowledgements

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Appendix A.

A.1. *Proof of Lemma 1*

We first list some definitions and calculations used in the proofs. We have previously defined

$$\xi = \sigma + \frac{1-\sigma}{M} = \frac{1+\sigma(M-1)}{M}.$$

This implies

$$\frac{\partial \xi}{\partial M} = -\frac{1-\sigma}{M^2};$$

$$\frac{\partial \xi}{\partial \sigma} = \frac{M-1}{M}.$$

The payoff of individual i writes as

$$V_i(e; \xi, M) = \xi[e_i]^\alpha - ce_i + v(e_{-i}, \xi, M).$$

Result 1. Every individual participating in contract (σ, M) invests

$$e_i = e(\xi) \equiv \left(\frac{\alpha}{c}\xi\right)^{1/1-\alpha}$$

Result 1 follows from the maximization of V_i with respect to $e_i \in [0, 1]$. The function $e(\xi)$ is increasing in ξ, and hence strictly decreasing in M whenever $0 \leq \sigma < 1$. This proves Lemma 1.

A.2. Proof of Lemma 2

Define

$$V(\xi, M) = \mu + [e(\xi)]^{\alpha} - ce(\xi) - \frac{\rho\nu}{2}\left(\xi^2 + \frac{(1-\xi)^2}{M-1}\right).$$

Note that

$$V_{\xi} \equiv \frac{\partial V}{\partial \xi} = \left(\frac{\alpha}{c^{\alpha}}\right)^{(1/(1-\alpha))} \frac{1}{1-\alpha}\xi^{(2\alpha-1)/(1-\alpha)}(1-\xi) - \rho\nu\sigma. \tag{A.1}$$

$$V_{\sigma} \equiv \frac{\partial V}{\partial \sigma} = \frac{M-1}{M}V_{\xi}; \tag{A.2}$$

$$V_M \equiv \frac{\partial V}{\partial M} = \frac{1}{M^2}\left(\frac{\rho\nu}{2}(1-\sigma)^2 - V_{\xi}(1-\sigma)\right). \tag{A.3}$$

Some properties of V can now be summarized.

Result 2. Let $Y(\xi) \equiv (\alpha/c^{\alpha})^{1/(1-\alpha)}(1/(1-\alpha))\xi^{(2\alpha-1)/(1-\alpha)}(1-\xi) = V_{\xi} + \rho\nu\sigma$.

1. $Y(1) = 0$
2. $\xi \in [1/M, 1) \Rightarrow Y(\xi) > 0$;
3. $Y(\xi)$ is monotonically declining whenever $0 < \alpha \leq 1/2$
4. $Y(\xi)$ is bounded above whenever $1/2 < \alpha < 1$:

$$Y(\xi) \leq \eta \equiv \max_{\xi\in[0,1]} Y(\xi) = \alpha(c)^{-\alpha/(1-\alpha)}(2\alpha-1)^{(2\alpha-1)/(1-\alpha)} < \infty.$$

Result 2 follows from properties of the function, and that $x^{\gamma}(1-x)$ attains a maximum at $x = \gamma/(1+\gamma)$ whenever $\gamma > 0$.

Result 3. V is increasing in M at $M = 1$.

Eqs. (A.1)–(A.3) imply

$$V_M = \frac{1-\sigma}{M^2}\left(\rho\nu\frac{(1+\sigma)}{2} - Y(\xi)\right) \tag{A.4}$$

and $M = 1 \Rightarrow \xi = 1 \Rightarrow Y = 0$.

Result 4. V is quasi-concave in M whenever $0 < \alpha \leq 1/2$. It is increasing in M for each σ whenever $\rho\nu > 2Y(\sigma + (1-\sigma)/M)/(1+\sigma)$.

The result follows from (9) and Result 2, claim 3.

Result 5. Let $1/2 < \alpha < 1$. For each $\sigma \in [0, 1)$, V is strictly increasing in M whenever $\rho\nu > 2\eta$.

From (A.4), $V_M > 0$ whenever $\rho\nu(1+\sigma) > 2Y(\xi)$. $\sigma \geq 0 \Rightarrow \rho\nu(1+\sigma) \geq \rho\nu$. From Result 2, claim 4, $Y(\xi) \leq \eta$ whenever $\alpha \in (\frac{1}{2}, 1)$.

For Lemma 2, define γ_N such that

$$0 < \alpha \le \frac{1}{2} \Rightarrow \gamma_N = 2Y\left(\frac{1}{N}\right);$$

$$1 > \alpha > \tfrac{1}{2} \Rightarrow \gamma_N = 2\eta$$

and invoke Results 3 and 4, noting that γ_N is finite whenever N is.

A.3. Proof of Lemma 3

For each M, $V(\sigma, M)$ is continuous in $\sigma \in [0, 1]$. Hence, a maximum exists. Further,

$$\frac{\partial V}{\partial \sigma} = \frac{M-1}{M}[Y(\xi) - \rho\nu\sigma]$$

and is continuous on [0, 1]. Now

$$\frac{\partial V}{\partial \sigma}|_{\sigma=1} = -\nu\rho\frac{M-1}{M} < 0$$

whenever $M > 1$, implying $\sigma^* < 1$ if $\rho\nu > 0$. Similarly,

$$\frac{\partial V}{\partial \sigma}|_{\sigma=0} = \frac{M-1}{M}Y\left(\frac{1}{M}\right) > 0,$$

implying $\sigma^* > 0$. It follows that σ_M^* is an interior optimum and necessarily satisfies:

$$\frac{Y(1 + (M-1)\sigma_M^*/M)}{\sigma_M^*} = \rho\nu,$$

the first order condition for a maximum. Finally, monotonicity of σ_M^* when $\alpha \le 1/2$ follows from the fact that Y declines with σ.

A.4. Proof of Lemma 4

Equilibrium payoffs are:

$$V(\xi, M) = \left(\frac{\alpha\xi}{c}\right)^{\alpha/(1-\alpha)} - c\left(\frac{\alpha\xi}{c}\right)^{1/(1-\alpha)} - \frac{\rho\nu}{2}\left(\xi^2 + \frac{(1-\xi)^2}{M-1}\right).$$

Further,

$$\frac{\mathrm{d}V}{\mathrm{d}M} = \frac{\partial V}{\partial \xi}\frac{\mathrm{d}\xi}{\mathrm{d}M} + \frac{\partial V}{\partial M}.$$

The efficient sharing rule is interior, from Lemma 3, and satisfies

$$\frac{\partial V}{\partial \xi}\frac{\partial \xi}{\partial \sigma} = 0,$$

which requires $\partial V/\partial \xi = 0$. Substituting the previous one, for the efficient contract at each M,

$$\frac{\mathrm{d}V}{\mathrm{d}M} = \frac{\partial V}{\partial M} = \frac{\rho\nu(1-\xi)^2}{2(M-1)^2} > 0.$$

A.5. Proof of Proposition 1

1. Define $M_k^{-i} = \sum_{j\neq i:1}^{N} a_{ik}$. Individual i chooses $a_{ik} = 1$ only if

   $$V(\sigma_k, M_k^{-i} + 1) = \max[\ldots, V(\sigma_j, M_j^{-i} + 1), \ldots].$$

2. Let $M_k^{-i} = N - 1$ and $M_j^{-i} = 0$ for $j \neq k$. Then $a_{ik} = 1$ only if

   $$V(\sigma_k, N) \geq V_a.$$

   This establishes that $M_k = N$ is an equilibrium whenever this condition holds.
3. Suppose $\rho\nu \geq \gamma_N$ as specified. From Lemma 3, this implies

   $$(\boldsymbol{M}) \quad V(\sigma_k, M + 1) > V(\sigma_k, M)$$

   for each $\sigma \in [0, 1)$ and integer M between 1 and N. Suppose $M_0 \geq 1$ and $M_k \geq 1$ at a participation equilibrium. This implies $V_a \geq V(\sigma_k, M_k + 1)$, which contradicts $(\boldsymbol{M})$. Hence, $\max[M_k] \geq 1 \Rightarrow M_0 = 0$. Suppose $M_1 > 1$ and $M_2 \geq 1$, $M_0 = 0$. This is a participation equilibrium if

   $$V(\sigma_1, M_1) \geq V(\sigma_2, M_2 + 1);$$

   $$V(\sigma_2, M_2) \geq V(\sigma_1, M_1 + 1).$$

   From $(\boldsymbol{M})$,

   $$V(\sigma_2, M_2 + 1) > V(\sigma_2, M_2).$$

   These three inequalities imply $V(\sigma_1, M_1) > V(\sigma_1, M_1 + 1)$ which violates $(\boldsymbol{M})$.

A.6. Proof of Proposition 2.

Define $V_k(m) = V(\sigma_k, m)$ and $V_j(m) = V(\sigma_j, m)$.

1. From Lemma 2, $\rho\nu > \gamma_N$ implies that $V(\sigma, M)$ is strictly increasing in M whenever $\sigma \in [0, 1)$. The function

   $$\Delta_{kj}(m) = V_k(m) - V_j(N + 1 - m)$$

   is strictly increasing in m, and

   $$\Delta_{kj}(1) = V_a - V_j(N) < 0;$$

   $$\Delta_{kj}(N) = V_k(N) - V_a > 0.$$

There exists some M^* such that

$$m < M^* \Leftrightarrow \Delta_{kj}(m) < 0.$$

2. Consider the stochastic process defined by the transition equation

$$M_{t+1} = M_t + 1 \quad \text{with probability } b_\epsilon(M_t);$$

$$M_{t+1} = M_t - 1 \quad \text{with probability } d_\epsilon(M_t).$$

The invariant distribution must satisfy the balance conditions

$$P_\epsilon(m)d_\epsilon(m) = P_\epsilon(m-1)b_\epsilon(m-1) \Rightarrow \frac{P_\epsilon(m)}{P_\epsilon(m-1)} = \frac{b_\epsilon(m-1)}{d_\epsilon(m)}$$

for each $m = 1, \ldots, N$.
3. From the definitions.

$$m < M^* \Rightarrow \frac{b_\epsilon(m-1)}{d_\epsilon(m)} = \frac{N-m+1}{m}\frac{\epsilon}{1-\epsilon}$$

$$m \geq M^* \Rightarrow \frac{b_\epsilon(m-1)}{d_\epsilon(m)} = \frac{N-m+1}{m}\frac{1-\epsilon}{\epsilon}.$$

4. It follows that

$$\frac{P_\epsilon(N)}{P_\epsilon(0)} = \left(\frac{1-\epsilon}{\epsilon}\right)^{N-2(M^*-1)}.$$

Thus,

$$\lim_{\epsilon \to 0} P_\epsilon(N) > 0$$

if, and only if, $M^* \leq N/2 + 1$.
5. $N^+/2$ is the largest integer not exceeding $N/2 + 1$. Thus, stability of (σ_k, N) relative to σ_j is equivalent to $M^* \leq N^+/2 \Rightarrow \Delta_{kj}(N^+/2) \geq 0$. This completes the proof.

A.7. Proof of Proposition 3

Let

$$V\left(\sigma_k, \frac{N^+}{2}\right) = \max_\sigma V\left(\sigma, \frac{N^+}{2}\right).$$

Then,

$$\Delta_{kj}\left(\frac{N^+}{2}\right) \geq V\left(\sigma_k, \frac{N^+}{2}\right) - V\left(\sigma_j, \frac{N^+}{2}\right) \geq 0.$$

Further, $N + 1 - (N^+/2) = N^+/2$ whenever N is odd, implying $\Delta_{kj}(N^+/2) \geq 0$ for all σ_j if, and only if, $V(\sigma_k, N^+/2) = \max_\sigma V(\sigma, N^+/2)$.

A.8. Proof of Proposition 4

Suppose individuals can choose between K groups with rules $\sigma_1, \dots, \sigma_K$. Let

$$S_t = [\dots, m_{kt}, \dots]$$

be the participation profile at t. Birth and death rates for each group depend on the full state variable S_t:

$$d_{kt}(S_t) = \frac{m_{kt}}{N}(1-\epsilon) \quad \text{if } \min_{j\neq k}\Delta_{kj}(m_{kt}) < 0$$

$$d_{kt}(S_t) = \frac{m_{kt}}{N}\epsilon \quad \text{otherwise},$$

and similarly for b_t (there is a slight difference for accidental births, where we assume that any of the "wrong" groups can be joined with equal probability $\epsilon/(K-1)$). These are birth and death probabilities conditional on the state S_t. Let P_ϵ be the stationary distribution on S, and define the marginal probabilities

$$d_k(m) = \frac{\sum_{S:m_k=m} P_\epsilon(S) d_k(S)}{\sum_{S:m_k=m} P_\epsilon(S)},$$

and similarly for $b_k(m)$. As before, $P_\epsilon(m_k = m) = P_\epsilon(0)\prod_{m=1}^{M} b_k(m)/d_k(m)$.

Now suppose

$$(\forall i)\left[V\left(\sigma_k, \frac{N^+}{2}\right) \geq V\left(\sigma_i, N+1-\frac{N^+}{2}\right)\right].$$

This implies $b_k(m-1)/d_k(m) = \phi(m)\frac{1-\epsilon}{\epsilon}$ for all $m \geq N^+/2$, where $\phi(m)$ is a finite, positive, function of m. The contract (σ_k, N) must then be stable. The proof that $(\sigma^*_{N^+/2}, N)$ is globally stable is immediate.

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